Allies or Commitment Devices? 
A Model of Appointments to the Federal Reserve

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March 3, 2016

Abstract

We present a model of executive-legislative bargaining over appointments to independent central banks in the face of an uncertain economy with strategic economic actors. The model highlights the contrast between two idealized views of Federal Reserve appointments. In one view, politicians prefer to appoint conservatively biased central bankers to overcome credible commitment problems that arise in monetary policy. In the other, politicians prefer to appoint allies, and appointments are well described by the spatial model used to describe appointments to other agencies. Both ideals are limiting cases of our model, which depend on the level of economic uncertainty. When economic uncertainty is extremely low, politicians prefer very conservative appointments. When economic uncertainty increases, politicians’ prefer central bank appointees closer to their own ideal points. In the typical case, the results are somewhere in between: equilibrium appointments move in the direction of politician’s preferences but with a moderate conservative bias.

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In the United States, the process of Presidential appointment and Senate confirmation of Federal Open Market Committee (FOMC) members is an important avenue for political influence on monetary policy. Two theoretical traditions inform political scientists’ understanding of Federal Reserve appointments. The first tradition holds that politicians face incentives to \textit{ex ante} commit to low inflation policies and \textit{ex post} break those commitments to generate short-term improvements in real economic outcomes, \textit{e.g.}, growth and employment (Kydland and Prescott, 1977; Barro and Gordon, 1983). Thus, delegating monetary policy authority to a relatively conservative, independent central bank(er) can serve as a commitment device that allows politicians to credibly commit to low inflation policies thereby mitigating this time-inconsistency problem (Rogoff, 1985; Alesina and Summers, 1993). The second tradition assumes instead that politicians prefer to appoint bankers that agree with them on matters of monetary policy and represents this process using spatial models of appointments (Chang, 2001, 2003; Morris, 2000). These models are consistent with “the ally principle” which states that politicians seek agents that represent their preferences as closely as possible (Bendor and Meirowitz, 2004).

These two theoretical traditions contradict one another, yet both appear to be consistent with data in important ways. In favor of the credible commitment approach, comparative empirical evidence suggests that countries with independent central banks experience lower levels of inflation (Alesina and Summers, 1993). In favor of the ally approach, empirical studies show that preferences of appointed FOMC governors track those of their appointing parties. Though the theories seem inconsistent with each other, we think that they represent a realistic trade-off that politicians face when delegating monetary policy: delegate to an ally and suffer high inflation due to the commitment problem, or appoint conservatively biased central bankers and allow some policy drift.\footnote{These models are similar to those used for appointments to other agencies (Lewis, 2008) or to the judiciary (Binder and Maltzman, 2009).}

\footnote{See, for example, Adolph (2013); Havrilesky (1988, 1994, 1995); Havrilesky and Gildea (1992); Chappell, Havrilesky and McGregor (1993); Chappell, McGregor and Vermilyea (2004a,b). However, as Adolph (2013) discussed, this finding is also consistent with classic economic models.}

\footnote{Clark and Arel-Bundock (2013) illustrate a complementary trade-off from the central banker’s point-of-view: they argue Fed governors must weigh the preferences of their principals, which vary, in order to preserve institutional independence. This \textit{ex post} trade-off in monetary policy choices generates variation in what appears to be instrumentally-induced partisan bias of the bankers.}
We develop a model of executive-legislative bargaining over appointments to the Federal Reserve. In our model, the President nominates a central banker that is subject to veto by the Senate. Inflationary expectations in the market then adjust to account for the central banker, and then at some later point the central banker sets monetary policy in response to an economic shock. The nature of appointment bargaining depends on the level of economic uncertainty. In certain times, politicians of all stripes prefer delegation to conservative central bankers. In uncertain times, appointments are more politicized because politicians prefer to appoint allies.

Recent work has also emphasized the role of uncertainty in monetary policy and appointments. Ainsley (Forthcoming) presents a novel model of central bank appointments in which the ideology of the central bank appointee responds to economic uncertainty. Our model differs from Ainsley’s in two ways. First, uncertainty is conceptualized much differently in our model than in Ainsley. Economic uncertainty in our model generates discretion for the central banker to increase inflation above market expectations. In contrast Ainsley models a situation in which central bankers have no additional information about the economy and respond to uncertainty because of asymmetric risk preferences. Thus, our model is designed to highlight the effects of time inconsistency while Ainsley’s highlights the effect of central bankers’ attitudes toward risk. Second, we include a model of inter-institutional bargaining over central bankers, while Ainsley focuses on the optimal central banker for a single policymaker that holds a monopoly over appointments. Thus, our model complements Ainsley (Forthcoming) and provides an alternative mechanism through which economic uncertainty may affect appointments.

In the next section we present our model of central bank appointments. We then characterize how central banker ideology affects policy choice and economic outcomes. The following section analyzes optimal central bank appointments from the perspective of the President and Senate. The final analysis section presents the main results of the paper, which illustrate how the nature of the economic environment structures the political appointment process. The final section provides a substantive discussion of these results, including empirical implications, and concludes.  

All proofs of formal results are relegated to the appendix.
A Model of Appointments to the Federal Reserve

To formalize the appointment process to the Federal Reserve we develop a non-cooperative game with a President, a representative Senator, a Central Banker, and a Wage Setter. We define the preferences of the actors by relying on a set of familiar economic assumptions, which are similar to several papers in the central banks literature (e.g., Alesina, Roubini and Cohen, 1997; Keefer and Stasavage, 2003). The economy is characterized by a “natural rate of inflation,” denoted by \( \bar{y} \) and a slope parameter \( \alpha > 0 \) that quantifies the trade-off between unemployment and inflation. Unemployment is generated by the following function:

\[
y(\pi, w) = \bar{y} - \alpha(\pi - w) - \varepsilon,
\]

where \( \pi \) denotes the level of inflation, \( w \) is the nominal wage, and \( \varepsilon \) is a stochastic shock to output distributed around mean 0 with finite variance \( \sigma^2 \). Equation 1 captures the intuition from classic economic models: unemployment can be driven to an unnaturally low level if the rate of inflation is higher than the growth in nominal wages. Politicians have an incentive to strategically drive unemployment down by creating surprise inflation and therefore delegation to an independent central banker that is relatively conservative (inflation-averse) may be desirable to solve the credible commitment problem this incentive creates. The optimal level of central banker conservatism will hinge on these economic considerations.

In particular, a key parameter that drives many of the results that follow is the variability of economic shocks \( \sigma \), which we refer to as economic uncertainty or volatility. The higher is \( \sigma \), the more uncertain the agents are about economic outcomes. Exogenous increases in \( \sigma \) capture shocks to the economy due to economic crises, global commodity shocks (e.g., oil prices), and other similar, relatively unpredictable events. In contrast, lower values of \( \sigma \) represent relatively stable economic environments in which there is lower uncertainty about future inflation.

With the unemployment function in hand we now characterize the preferences of the President,
Senator, and Central Banker. These preferences are given by the following utility function,

$$u_i(\pi, y(\pi, w)) = -\pi^2 - b_i y^2 \text{ for } i \in \{P, S, C\}.$$  \hspace{1cm} (2)

Each actor has the same target rates of inflation and unemployment, which are normalized to 0, and differ only by the relative weights they place on inflation and unemployment represented by $b_i \geq 0$. We refer to $b_i$ as the actor $i$’s ideal point or monetary policy position where lower levels of $b_i$ denote a more conservative or inflation-averse ideal point whereas a higher $b_i$ denotes a liberal or more employment-focused ideal point.

The preferences of the Wage Setter are given by the following utility function,

$$u_W = -(\pi - w)^2.$$ \hspace{1cm} (3)

Equation 3 ensures that nominal wages will be set equal to expected inflation in equilibrium.

Prior to any actions there is a status quo Central Banker with ideal point $b_{SQ}$ who remains in office until replaced by a newly appointed Central Banker. We understand $b_{SQ}$ to represent the common understanding of how current members of the FOMC will implement monetary policy in the absence of a new appointment. The timing of the game, then, is as follows.

1. The President selects a nominee by proposing $b > 0$.
2. The Senator accepts or rejects the nominee:
   
   (a) If $S$ accepts the nominee, then $b_C = b$.
   
   (b) If $S$ rejects the nominee, then $b_C = b_{SQ}$.
3. The Wage Setter chooses $w$.
4. The output shock, $\varepsilon$, is realized.
5. The Central Banker sets the level of inflation, $\pi$. 

4
**Equilibrium Inflation and Unemployment**

We utilize subgame perfect Nash equilibrium (SPNE) as our solution concept, which we find through backward induction. In the final stage of the game the Central Banker sets the target rate of inflation, $\pi$. Thus, the Central Banker takes an action solving the following problem:

$$
\pi^*(b_C, w) = \arg \max_\pi \left[ -\pi^2 - b_C(\bar{y} - \alpha(\pi - w) - \varepsilon)^2 \right].
$$

Essentially, the Central Banker chooses the inflation rate that maximizes her expected utility. Solving this problem for the Central Banker leads to the following best response function,

$$
\pi^*(b_C, w) = \frac{b_C\alpha(w \alpha + \bar{y} - \varepsilon)}{b_C\alpha^2 + 1}.
$$

(4)

Notice that the best response $\pi^*(b_C, w)$ depends on $w$: the Wage Setter’s choice of wage level. Thus, to fully characterize the subgame following approval (or rejection) of the central bank appointment we solve the Wage Setter’s problem, given by:

$$
w^*(b_C) = \arg \max_w \left[ -E[(w - \pi^*(b_C))^2] \right].
$$

(5)

At the solution, wages are simply set equal to the expected level of inflation. Thus, the wage rate in equilibrium is $w^*(b_C) = b_C\bar{y}$. We can substitute this expression into the solution for $\pi^*(b_C, w)$, which yields the following result.

**Lemma 1.** In equilibrium, a Central Banker with monetary policy ideal point $b_C$ sets the target level of inflation according to the following equation:

$$
\pi^*(b_C) = \frac{\alpha b_C \left( \alpha^2 \bar{y} b_C + \bar{y} - \varepsilon \right)}{\alpha^2 b_C^2 + 1}.
$$

(6)

The pair $(w^*(b_C), \pi^*(b_C))$ is an equilibrium to the subgame involving the Wage Setter’s and Central Banker’s decisions. The equilibrium to this subgame depends on $b_C$ and therefore induces
preferences for the President and Senator regarding the optimal central bank appointment. We can now characterize how economic outcomes are affected according to the preferences of the Central Banker. Proposition 1 describes how these economic outcomes—inflation and unemployment—depend on these preferences.

**Proposition 1.** *The preferences of the Central Banker affect economic outcomes as follows:*

1. *Expected inflation* is both higher and more variable as the Central Banker becomes more liberal.

2. *Expected unemployment* levels are independent of the Central Banker but become less variable as the Central Banker becomes more liberal.

The conclusions in Proposition 1 are straightforward. Central Bankers who place less emphasis on low inflation relative to low unemployment will be more tempted to generate inflation, therefore expected inflation is higher for these actors. Moreover, while conservative Central Bankers tend to moderate the effects of output shocks, liberal Central Bankers utilize shocks to generate lower unemployment, thereby creating higher levels of variability in the rate of inflation.

The inability of the Central Banker to raise expected unemployment is a result of the standard rational expectations argument: wage setters anticipate the effect of a liberal central banker on expected inflation and incorporate this information into wage contracts, so that the average effect of monetary policy on unemployment is null. However, since the central banker is, at times, able to take advantage of output shocks, employment is more stable for more liberal central bankers.

**Equilibrium Central Bank Appointments**

With the equilibrium behavior of the Wage Setter, the Central Banker, and how the preferences of a given appointed Central Banker affect inflation and unemployment in hand we can now finish constructing the SPNE of the appointments game by characterizing the equilibrium appointment strategy of the President and the equilibrium approval strategy of the Senator. Given the strategies characterized in the previous section and the fact that both the President and the Senator must act
prior to the realization of the output shock $\varepsilon$, their equilibrium strategies are based on expected utility. In particular, the expected (squared) level of inflation, as characterized in Lemma 1, is given by:

$$\pi^*(b_C)^2 = \text{Var}[\pi^*(b_C)] + \mathbb{E}[\pi^*(b_C)] = \frac{\alpha^2 \sigma^2 b_C^2}{(\alpha^2 b_C + 1)^2} + b_C^2 y^2 \alpha^2.$$  

Similarly, the expected (squared) level of unemployment is given by:

$$\mathbb{E}[y(\pi, w)^2] = \frac{\sigma^2}{(\alpha^2 b_C + 1)^2} + \bar{y}^2.$$  

Substituting these expectations into the utility functions for the President and the Senator yields the following expected utility:

$$U_i(b_C) = \mathbb{E}(u_i(\pi, y(\pi, w)|b_C) = -\frac{(\alpha^2 b_C^2 + b_i) \left(\bar{y}^2 \left(\alpha^2 b_C + 1\right)^2 + \sigma^2\right)}{(\alpha^2 b_C + 1)^2}, i \in \{S, P\}. \quad (7)$$

From this expected utility expression, we can conclude that there is a unique ideal appointee for the President and for the Senator. Furthermore, the ideal appointee for each actor is strictly more conservative than that actor. Thus, induced preferences over appointees take on a spatial structure, but it is one in which neither of the actors prefer to appoint an agent that exactly represents their own interests. The preferences of the actors therefore violate the ally principle, which holds in a wide range of other models of delegation (Bendor, Glazer and Hammond, 2001; Bendor and Meirowitz, 2004).\(^5\)

**Proposition 2.** The President and Senator each have an “ideal” Central Bank appointee and both actors prefer to delegate to a Central Banker who is strictly more conservative than themselves.

A best response for the Senator involves accepting nominees for appointment such that the

\(^5\)We use the term *ally principle* in the same way as Bendor and Meirowitz (2004), who take the term to mean that “the boss picks the most ideologically similar agent as delegatee.” We do not mean to imply, however, that violation of the ally principle in this case runs counter to the logic of their model. In fact, a departure from the ally principle is to be expected when interpreting this model in light of Bendor and Meirowitz’s (2004) results since the agent does not fully control unemployment. Thus, when we discuss our model in relation to the ally principle, we are comparing our results to other theoretical models (e.g. Chang (2001, 2003)) in which the assumptions are consistent with the ally principle.
Senator’s expected utility given the appointee’s monetary policy ideology (weakly) outweighs the utility the Senator would receive from rejecting the nominee and retaining the status quo, i.e., \( U_S(b) \geq U_S(b^{SQ}) \). This is akin to the Senator using a threshold acceptance strategy, which can also be represented by an equivalent “acceptance set.” The Senator will approve a Central Bank nominee with monetary policy ideology \( b \) if and only if the nominee is in the Senator’s acceptance set \( A_S \) (i.e., \( b \in A_S = \{ b : U_S(b) \geq U_S(b^{SQ}) \} \)). The President knows what nominees are acceptable to the Senator and which are not and therefore maximizes his expected utility subject to the constraint provided by \( A_S \). Since \( b^{SQ} \in A_S \) and the President is indifferent between choosing a status quo nominee and being rejected, we assume that the President chooses \( b^{SQ} \) in the event that no element of \( A_S \) is preferred to the status quo. The full SPNE to this game therefore is the solution to a standard spatial bargaining game (Romer and Rosenthal, 1978; Ferejohn and Shipan, 1990) with these induced preferences. In the next section, we characterize the predictions that arise from the equilibrium to the game.

**Economic Uncertainty and the Optimal Central Banker**

When are central bank appointments characterized by a tendency to appoint allies? When do politicians instead use central bankers as commitment devices to keep inflation low? To answer these questions we provide a series of results that speak directly to how politicized the appointments process is conditional on the economic environment. The first result concerns the relationship between the positions of the President and Senator and that of the equilibrium nominee. Although Proposition 2 predicts that the President and Senator both prefer a nominee more conservative than themselves the following result confirms that the ideal position of nominees is positively correlated with the positions of the President and Senator.

**Proposition 3.** The ideological position of the equilibrium Central Bank appointment is weakly increasing in the positions of the President and the Senator.

The President and the Senator prefer a central bank appointee that is more conservative than
they are (Proposition 2), but how conservative the equilibrium appointment will be relative to
the political actors bargaining over the appointment depends on the ideological positioning of the
President and Senator. As Adolph (2013) has noted, the prediction of Proposition 3 holds both in
the spatial models of Chang (2001, 2003) and in the economic models like that of Rogoff (1985).
Notice, however, that Proposition 3 merely shows that as the President and the Senator become
more liberal, the Central Banker appointed in equilibrium could become more liberal as well. The
main insight here is that as the ideal points of the actors bargaining over the appointment move
toward more liberal monetary policy preferences, the ideal appointment becomes (weakly) more
liberal. This result does not, in itself, provide insight into when and why the appointments process
is more or less politicized. Combined with the following results, however, we do begin to generate
insight into the effect of economic uncertainty on the political appointment process.

As described informally in the previous section, the relationship between politicians’ pref-
ferences and equilibrium appointees observed in this model depends critically on uncertainty in
the economy. Since large variations in unemployment can occur with very conservative central
bankers, risk averse politicians will strike a balance between delegating to conservative central
bankers and appointing allies. This dynamic leads to the following result.

**Proposition 4.** In equilibrium, Central Bank appointments become more liberal as the economy
becomes more volatile.

Combined with the previous result, Proposition 4 presents two limiting cases for equilibrium
appointments. As economic uncertainty approaches zero, i.e., \( \sigma \to 0 \), appointments become per-
fectly conservative. All politicians prefer appointees that are fully focused on inflation and do not
care at all about unemployment. Conversely, as economic uncertainty rises, i.e., \( \sigma \to \infty \), indi-
vidual behavior resembles a politicized appointment process in which both the President and the
Senator prefer ideological allies. The second observation follows from the fact that, for any finite
\( \sigma \), Proposition 2 implies that politicians prefer appointees that are strictly more conservative than
themselves. Thus, since preferred appointments are strictly increasing in economic uncertainty,
\( \sigma \), the politician’s ideal appointment approaches their own position from below as this uncertainty
becomes large. The combination of the limiting cases provided by Propositions 3 and 4 lead to the main insight of this paper.

**Proposition 5.** As $\sigma$ goes to zero, the game converges to one in which the President and Senator mutually prefer a perfectly conservative central banker ($b_c = 0$). As $\sigma$ goes to infinity, the game converges to one in which the President and Senator each prefer central bankers with preferences identical to their own ($b_C = b_i$, $i \in \{P, S\}$).

Proposition 5 provides a testable restriction on the strategies of the President and the Senator that can be contrasted with traditional spatial models of central bank appointments (e.g., Chang, 2001, 2003; Morris, 2000). In particular, Proposition 5 suggests that, in less volatile economic environments, the political actors involved in bargaining over the appointed central banker—the President and the Senator—will prefer an appointee that more closely resembles their respective ideal monetary policy ideologies while in other, more volatile, environments they will be more aligned in their collective preference for a conservative central banker. Thus, when uncertainty is high the results predict that we should observe higher levels of conflict between ideologically divergent executives and legislative actors (as in Chang, 2003; Morris, 2000, for example). When uncertainty is lower we should observe a more consensual process characterized by a common value, relatively conservative, appointment (as in Rogoff, 1985, for example).

Taken together, Propositions 3-5 describe a link between economic conditions and political conflict over Fed appointments. To conceptualize the predictions of our model, consider the effects of exogenous shocks to economic uncertainty. Such shocks may arise from economic crises, supply shocks to commodities such as oil, natural disasters, or any number of other events. The theory predicts that positive shocks to economic uncertainty will cause monetary nominations to become more hotly contested, with liberal politicians preferring much more doveish appointees relative to their conservative colleagues. In contrast, periods of low economic uncertainty should generate more consensus on nominees, with liberal and conservative politicians mostly agreeing on relatively conservative nominees.
Conclusion

Theoretical work on Federal Reserve appointments has followed two different traditions which appear to contradict one another: the purely spatial approach assumes that politicians wish to appoint allies and the rational expectations tradition predicts that politicians will appoint biased central bankers in order to commit to low inflation policies. We show that the predictions of each theoretical tradition can be recovered from a simple political economic theory of appointments. Our model predicts that credible commitment incentives will dominate in times of economic certainty, when politicians of all stripes will agree on relatively conservative central bankers. In times of crisis or high uncertainty, we predict that central bank appointments will converge toward the predictions of purely spatial models.

In addition, some of the insights of the model may extend beyond the domain of monetary policy. It has also been observed in other domains that commitment problems may generate deviations from the ally principle. For instance, Bertelli and Feldmann (2007) show that, when policy outcomes are the result of bureaucrats negotiating with constituents, Presidents may prefer appointees with preferences that offset those of organized interests in the constituency. Gailmard and Hammond (2011) make a similar point about delegation to biased committee members in the presence of intercameral bargaining. Both environments are similar to our monetary policy environment in that delegation to a biased (relative to the principal) agent solves a commitment problem for the principal that allows her to obtain a superior outcome. Our main result suggests that more explicitly considering the role of policy uncertainty in these processes could generate a more nuanced understanding of when commitment problems should generate large differences in delegation behavior.

References


**Appendix**

**Proof of Lemma 1.** Recall the Central Banker solves the following problem:

$$\max_{\pi} \left[-\pi^2 - b_C (\bar{y} - \alpha (\pi - w) - \varepsilon)^2\right].$$

The first order condition for a maximum is:

$$-2\pi - b_C \alpha (\bar{y} - \alpha (\pi - w) - \varepsilon) = 0.$$  

Solving the first order condition and substituting in $w^*(b_C)$ yields the best response given by Equation 6. The second order condition is met since $u_C$ is concave. Therefore, this is a best response for the Central Banker and $\pi^*(b_C)$ is the inflation rate chosen in a SPNE. ■
Proof of Proposition 1. Claim (1) follows easily from the derived strategies. The expected value of inflation is \( b \bar{y} \alpha \) which is clearly increasing in \( b_C \). The variance of inflation is

\[
\frac{\alpha^2 \sigma^2 b_C^2}{(\alpha^2 b_C + 1)^2}
\]  

(8)

which is increasing in \( b_C \) since

\[
\frac{\partial}{\partial b_C} \frac{\alpha^2 \sigma^2 b_C^2}{(\alpha^2 b_C + 1)^2} = \frac{2 \alpha^2 \sigma^2 b_C}{(\alpha^2 + 1)^2} > 0.
\]  

(9)

Since \( E(w^*(b_C) - \pi^*(b_C)) = 0 \) for any \( b_C \) and \( E(\varepsilon) = 0 \), the expected value of unemployment is simply \( \bar{y} \) for all \( b_C \). The variance of \( y(\pi^*(b_C), w^*(b_C)) \) is

\[
\frac{\sigma^2}{(\alpha^2 b_C + 1)^2}
\]  

(10)

which is strictly decreasing in \( b_C \), proving claim (2). ■

Proof of Proposition 2. We will show that there exists one local maximum to \( U_i \) which lies in the open interval \((0, b_i)\) for any \( b_i > 0 \). The first order condition is

\[
\frac{\partial U_i}{\partial b_C} = \frac{2 \alpha^2 (\alpha^2 b_C^2 + b_i) (\bar{y}^2 (\alpha^2 b_C + 1)^2 + \sigma^2) - 2 \alpha^2 \bar{y}^2 (\alpha^2 b_C^2 + b_i)}{(\alpha^2 b_C + 1)^3}
\]

\[
= \frac{2 \alpha^2 b_C (\bar{y}^2 (\alpha^2 b_C + 1)^2 + \sigma^2)}{(\alpha^2 b_C + 1)^2}
\]

\[
= -\frac{2 \alpha^2 (\alpha^6 \bar{y}^2 b_C^4 + 3 \alpha^4 \bar{y}^2 b_C^3 + 3 \alpha^2 \bar{y}^2 b_C^2 + b_C (\bar{y}^2 + \sigma^2) - \sigma^2 b_i)}{(\alpha^2 b_C + 1)^3} = 0
\]

Since the denominator and the term \( 2 \alpha^2 \) must be strictly greater than zero, the solution to the first order condition must be a root to the quartic equation in the numerator:

\[
\alpha^6 \bar{y}^2 b_C^4 + 3 \alpha^4 \bar{y}^2 b_C^3 + 3 \alpha^2 \bar{y}^2 b_C^2 + b_C (\bar{y}^2 + \sigma^2) - \sigma^2 b_i = 0.
\]
We can re-write this expression as

\[ b_C = g(b_C) = b_i \cdot \frac{\sigma^2}{y^2(\alpha^2 b_C + 1)^3 + \sigma^2}. \]  

(11)

Thus, the solution to our first-order condition is a fixed point of the function \( g(\cdot) \). Since the fraction in Equation 11 is always less than 1, \( g \) is bounded above by \( b_i \). Since \( g \) is also strictly positive, \( g \) is a continuous function mapping the interval \([0, b_i]\) onto itself. By Brouwer’s fixed point theorem, there exists a fixed point of \( g \) on this interval. By the arguments above, the fixed point is on the interior of this interval. Since \( U_i \) is strictly concave, this fixed point is a maximum of \( U_i \) and is unique. ■

**Lemma 2.** For any parameter \( \theta \), if \( U_S \) satisfies increasing differences\(^6\) for \((b_C, \theta)\), \( A_S \) is weakly increasing in \( \theta \) in the strong set order.\(^7\)

**Proof:** Let \( \bar{\theta} > \theta \) and denote \( \bar{A}_S \) and \( A_S \) denote the set \( A_S \) when the parameter \( \theta \) is set equal to \( \bar{\theta} \) and \( \theta \), respectively. We consider three cases: (1) there is no \( b'_C \neq b^{SO} \) such that \( U_i(b'_C, \theta) = U_i(b^{SO}, \theta) \); (2) there exists such a \( b'_C \), and \( b'_C > b^{SO} \); and (3) there exists such a \( b'_C \), and \( b'_C > b^{SO} \). If there does not exist \( b'_C \) such that \( U_i(b'_C, \theta) = U_i(b^{SO}, \theta) \), then \( A_S = [0, b^{SO}] \). Since \( \bar{A}_S \) must be an interval including \( b^{SO} \), we have \( \bar{A}_S \geq_s A_S \).

Now assume there exists a \( b'_C \) such that \( U_i(b'_C, \theta) = U_i(b^{SO}, \theta) \). If \( b'_C > b^{SO} \), then \( A_S = [b'_C, b^{SO}] \). In this case, we must show that, for any \( b''_C < b'_C, b''_C \notin \bar{A}_S \). By increasing differences,\(^8\)

\[
U_S(b'_C, \theta) - U_S(b''_C, \theta) > 0 \Rightarrow U_S(b'_C, \overline{\theta}) - U_S(b''_C, \overline{\theta}) \geq 0
\]

\[
U_S(b^{SO}, \theta) - U_S(b'_C, \theta) = 0 \Rightarrow U_S(b^{SO}, \overline{\theta}) - U_S(b'_C, \overline{\theta}) \geq 0.
\]

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\(^6\)A function \( f(x, \theta) \) satisfies **increasing differences** for \((x, \theta)\) if the incremental return, \( f(x, \cdot) - f(x', \cdot) \), is weakly increasing in the parameter, \( \theta \).

\(^7\)The strong set order is denoted \( \geq_s \). When the choice set is a subset of the real line as in this paper and \( S^* \) and \( S^0 \) are intervals, we will have \( S^* \geq_s S^0 \) provided that the end-points of \( S^* \) are greater than or equal to the end-points of \( S^0 \).

\(^8\)This statement is an application of the single-crossing condition, which is implied by increasing differences.
Hence, \( U_S(b^S, \overline{\theta}) \geq U_S(b'_C, \overline{\theta}) > U_S(b''_C, \overline{\theta}) \), which implies that \( b''_C \not\in \overline{A_S} \). Since \( \overline{A_S} \) must be an interval including \( b^S \), this implies that \( \overline{A_S} \geq s \).

Finally, if \( b'_C > b^S \), then \( \overline{A_S} = [b^S, b'_C] \). In this case, we must show that \( b'_C \in \overline{A_S} \). By increasing differences,

\[
U_S(b'_C, \overline{\theta}) - U_S(b^S, \overline{\theta}) = 0 \Rightarrow U_S(b'_C, \overline{\theta}) - U_S(b^S, \overline{\theta}) \geq 0
\]

which implies that \( b'_C \in \overline{A_S} \). The argument in Case 2 shows that there is no point in \( \overline{A_S} \) smaller than \( b^S \). □

we will use the following result, which follows from the monotone selection theorem of Milgrom and Shannon (1994):\(^9\)

**Monotone Selection Theorem (Milgrom and Shannon (1994))**: Let \( X \subseteq \mathbb{R} \) be the set of all possible choices of \( x \) and \( \Theta \subseteq \mathbb{R} \) be the set of possible values of a parameter \( \theta \). Let \( f : X \times \Theta \rightarrow \mathbb{R} \). If \( S : \Theta \rightarrow 2^X \) is nondecreasing and \( f \) satisfies the single-crossing condition for \( (x, \theta) \), then every selection \( x^*(\theta) \) from \( \arg \max_{x \in S(\theta)} f(x, \theta) \) is monotone nondecreasing in \( \theta \).

**Proof of Proposition 3**: Using the theorem of Milgrom and Shannon (1994), it is sufficient to show that \( U_P(b) \) satisfies single-crossing for \( (b, b_P) \) and for \( (b, b_S) \) and that \( A_S \) is nondecreasing in \( b_P \) and \( b_S \). Since \( b_S \) does not enter \( U_P(b) \), so \( U_P(b) \) satisfies single-crossing for \( (b, b_S) \) since the cross-partial derivative of \( U_P(b) \) with respect to \( b \) and \( b_S \) is zero. Similarly, since \( U_S(b_C) \) does not depend on \( b_P \), \( A_S \) is non-decreasing in \( b_P \). To show that \( U_P(b) \) satisfies single-crossing for \( (b, b_P) \), note that

\[
\frac{\partial U_P}{\partial b_C \partial b_P} = \frac{2a^2 \sigma^2}{(\alpha^2 b_C + 1)^3} > 0,
\]

which shows that the increasing differences condition is satisfied, implying that the single-crossing is met. Since, by the same argument, \( U_S(b_C) \) satisfies single-crossing for \( (b_C, b_S) \), Lemma 2 establishes that \( A_S \) is nondecreasing in \( b_S \), completing the proof. □

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\(^9\)This is Theorem 4’ in Milgrom and Shannon (1994). Their result is stated for more general multidimensional problems and requires introducing concepts that we have not defined above, so we restate it for the special case of one-dimensional choice sets and parameters and using the notation introduced above.
Proof of Proposition 4. By the arguments above, we need to show that $U_i$ satisfies single crossing for $(b_C, \sigma)$ when $b_C$ is in the interval $(0, b_i)$. Note that

$$\frac{\partial U_i}{\partial b_C \partial \sigma} = \frac{4\alpha^2 \sigma (b_i - b_C)}{(\alpha^2 b_C + 1)^3},$$

which is strictly positive given that $b_C < b_i$. Thus, $U_i$ satisfies increasing differences (therefore single-crossing) for $(b_C, \sigma)$, completing the proof. ■

Proposition 5. By Equation 11 in the proof of Proposition 2, the ideal appointment of each agent $i$ is equal to

$$b_i \cdot \frac{\sigma^2}{\bar{y}^2 (\alpha^2 b_C + 1)^3 + \sigma^2}.$$

We have

$$\lim_{\sigma \to 0} b_i \cdot \frac{\sigma^2}{\bar{y}^2 (\alpha^2 b_C + 1)^3 + \sigma^2} = 0$$

and

$$\lim_{\sigma \to \infty} b_i \cdot \frac{\sigma^2}{\bar{y}^2 (\alpha^2 b_C + 1)^3 + \sigma^2} = b_i \lim_{\sigma \to \infty} \frac{\sigma^2}{\bar{y}^2 (\alpha^2 b_C + 1)^3 + \sigma^2} = b_i.$$

Furthermore, Proposition 4 shows that every agent’s ideal appointment is increasing in $\sigma$. Thus, as $\sigma$ goes to zero, the game approaches one with perfectly conservative appointments and as $\sigma$ gets large the game monotonically approaches the spatial model of appointments in which agents seek appointees at their own ideal point. ■